EXERCICIS SOBRE CÀLCUL DE DERIVADES I

- **Si f(x) = k amb k ∈ R → f'(x) = 0**

Exemples : $(3)'=0$

$f(x)=\pi \rightarrow f'(x)=(\pi)'=0$

$y=\frac{2}{3} \rightarrow y'=\left(\frac{2}{3}\right)'=0$

- **Si f(x) = x^n → f'(x) = n · x^{n-1}**

Exemples : $(x^3)'=3x^2$

$(x^5)'=5x^4$

$(x)'=(x^1)'=1x^0=1$

$y=x^{-2} \rightarrow y'=(x^{-2})'=-2x^{-2-1}=-2x^{-3}$

$\left(\frac{1}{x^3}\right)'=(x^{-3})'=-3x^{-3-1}=-3x^{-4}=-3\cdot\frac{1}{x^4}=\frac{-3}{x^4}$

$f(x)=\frac{1}{x^6} \rightarrow f'(x)=\left(\frac{1}{x^6}\right)'=(x^{-6})'=-6x^{-7}=-6\cdot\frac{1}{x^7}=\frac{-6}{x^7}$

$(\sqrt{x})'=\left(x^{\frac{1}{2}}\right)'=\frac{1}{2}x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$

$(\sqrt[3]{x^5})'=\left(x^{\frac{5}{3}}\right)'=\frac{5}{3}x^{\frac{5}{3}-1}=\frac{1}{2}x^{\frac{2}{3}}=\frac{1}{2}\sqrt[3]{x^2}$

$y=\frac{1}{\sqrt[5]{x^6}} \rightarrow y'=\left(x^{-\frac{6}{5}}\right)'=-\frac{6}{5}x^{-\frac{6}{5}-1}=-\frac{6}{5}x^{-\frac{11}{5}}=-6\cdot\frac{1}{\sqrt[5]{x^{11}}}=\frac{-6}{\sqrt[5]{x^{11}}}$

- **Si $f(x)=k\cdot g(x)$ → $f'(x)=k\cdot g'(x)$**

Exemples : $\left(5x^6\right)'=5\cdot\left(x^6\right)'=5\cdot 6x^5=30x^5$

$f(x)=-6x^3\rightarrow f'(x)=\left(-6x^3\right)'=-6\cdot\left(x^3\right)'=-6\cdot 3x^2=-18x^2$

$\left(10x^2\right)'=10\cdot\left(x^2\right)'=10\cdot 2x^1=20x$

$y=\dfrac{5}{x^2}\rightarrow y'=\left(\dfrac{5}{x^2}\right)'=\left(5x^{-2}\right)'=5\cdot(-2)x^{-2-1}=-10x^{-3}=-\dfrac{10}{x^3}$

$\left(5\cos x\right)'=5\cdot\left(\cos x\right)'=5\cdot(-\sin x)=-5\sin x$

$\left(\dfrac{2}{7}\ln x\right)'=\dfrac{2}{7}\cdot(\ln x)'=\dfrac{2}{7}\left(\dfrac{1}{x}\right)=\dfrac{2}{7x}$

$\left(\sqrt{3}arctgx\right)'=\sqrt{3}\cdot\left(arctgx\right)'=\sqrt{3}\cdot\left(\dfrac{1}{1+x^2}\right)=\dfrac{\sqrt{3}}{1+x^2}$

$\left(5\cdot 3^x\right)'=5\cdot\left(3^x\right)'=5\cdot 3^x\cdot\ln 3=5\cdot(\ln 3)\cdot 3^x$

- **Si $f(x)=h(x)\pm g(x)$ → $f'(x)=h'(x)\pm g'(x)$**

Exemples : $\left(5x^3+x^4\right)'=\left(5x^3\right)'+\left(x^4\right)'=15x^2+4x^3$

$\left(10x^5-6x+5\right)'=\left(10x^5\right)'-\left(6x\right)'+\left(5\right)'=50x^4-6+0=50x^4-6$

$f(x)=-3x^4+x^{-2}+5x\rightarrow f'(x)=\left(-3x^4+x^{-2}+5x\right)'=$

$\left(-3x^4\right)'+\left(x^{-2}\right)'+\left(5x\right)'=-12x^3-2x^{-3}+5$

$\left(5e^x-x^4\right)'=\left(5e^x\right)'-\left(x^4\right)'=5e^x-4x^3$

$\left(\arcsin x+5x^4\right)'=\left(\arcsin x\right)'+\left(5x^4\right)'=\dfrac{1}{\sqrt{1-x^2}}+20x^3$

$y=\sqrt{x}-\log_2 x\rightarrow y'=\left(x^{\frac{1}{2}}\right)'-\left(\log_2 x\right)'=\dfrac{1}{2}x^{-\frac{1}{2}}-\dfrac{1}{x\ln 2}=\dfrac{1}{2\sqrt{x}}-\dfrac{1}{x\ln 2}$

- **Si $f(x)=h(x)\cdot g(x) \rightarrow f'(x)=h'(x)\cdot g(x)+h(x)\cdot g'(x)$**

Exemples :

$$\left(5x^3\cdot\sin x\right)'=\left(5x^3\right)'\cdot\sin x+5x^3\left(\sin x\right)'=15x^2\sin x+5x^3\cos x$$

$$\left(x^4e^x\right)'=\left(x^4\right)'\cdot e^x+x^4\cdot\left(e^x\right)'=4x^3e^x+x^4e^x=e^x(4x^3+x^4)$$

$$f(x)=2^x\cdot\cos x\rightarrow f'(x)=\left(2^x\cdot\cos x\right)'=\left(2^x\right)'\cdot\cos x+2^x\left(\cos x\right)'=$$
$$2^x\ln 2\cos x+2^x(-\sin x)=2^x\ln 2\cos x-2^x\sin x$$

$$y=\sqrt{x}\log_2 x\rightarrow y'=\left(x^{\frac{1}{2}}\right)'\cdot\log_2 x-\sqrt{x}\left(\log_2 x\right)'=$$
$$\frac{1}{2}x^{-\frac{1}{2}}\cdot\log_2 x-\sqrt{x}\frac{1}{x\ln 2}=\frac{\log_2 x}{2\sqrt{x}}-\frac{\sqrt{x}}{x\ln 2}$$

$$\left((5x^3-10x^4)\cdot\sin x\right)'=(5x^3-10x^4)'\cdot\sin x+(5x^3-10x^4)\left(\sin x\right)'=$$
$$\left(15x^2-40x^3\right)\sin x+(5x^3-10x^4)\cos x$$

$$\left(\sqrt[5]{x^2}\cdot arc\cot x\right)'=(\sqrt[5]{x^2})'\cdot arc\cot x+(\sqrt[5]{x^2})\left(arc\cot x\right)'=$$
$$\left(\frac{2}{5\sqrt[5]{x^3}}\right)arc\cot x+(\sqrt[5]{x^2})\frac{-1}{1+x^2}=\frac{2arc\cot x}{5\sqrt[5]{x^3}}-\frac{\sqrt[5]{x^2}}{1+x^2}$$

- **Si $f(x)=\dfrac{h(x)}{g(x)} \rightarrow f'(x)=\dfrac{h'(x)\cdot g(x)-h(x)\cdot g'(x)}{\left[g(x)\right]^2}$**

Exemples :

$$\left(\frac{x^2+3x}{5x}\right)'=\frac{(x^2+3x)'\cdot 5x-(x^2+3x)\cdot(5x)'}{(5x)^2}=\frac{(2x+3)\cdot 5x-(x^2+3x)\cdot 5}{25x^2}=$$
$$\frac{5x^2}{25x^2}=\frac{1}{5}$$

$$y=\frac{x^4}{e^x}\rightarrow y'=\left(\frac{x^4}{e^x}\right)'=\frac{\left(x^4\right)'\cdot e^x-x^4\cdot\left(e^x\right)'}{\left(e^x\right)^2}=\frac{4x^3e^x-x^4e^x}{e^{2x}}=\frac{e^x(4x^3-x^4)}{e^{2x}}=$$

$$\frac{4x^3-x^4}{e^x}$$

$$f(x)=\frac{x+2}{x-3}\rightarrow f'(x)=\left(\frac{x+2}{x-3}\right)'=\frac{1\cdot(x-3)-(x+2)\cdot 1}{(x-3)^2}=\frac{-5}{(x-3)^2}$$

$$\left(\frac{3\cos x}{5x^7}\right)'=\frac{\left(3\cos x\right)'\cdot 5x^7-3\cos x\cdot(5x^7)'}{(5x^7)^2}=\frac{-3\sin x\cdot 5x^7-3\cos x\cdot 35x^6}{25x^{14}}=$$

$$\frac{-15x^6(x\sin x+7\cos x)}{25x^{14}}=-\frac{3(x\sin x+7\cos x)}{5x^8}$$

$$\left(\frac{3x^4-2x}{5\cdot 3^x}\right)'=\frac{\left(3x^4-2x\right)'\cdot 5\cdot 3^x-\left(3x^4-2x\right)\left(5\cdot 3^x\right)'}{\left(5\cdot 3^x\right)^2}=\frac{\left(12x^3-2\right)5\cdot 3^x-\left(3x^4-2x\right)5\cdot 3^x\ln 3}{25\cdot(3^x)^2}=$$

$$\frac{5\cdot 3^x\left[\left(12x^3-2\right)-\left(3x^4-2x\right)\ln 3\right]}{25\cdot(3^x)^2}=\frac{12x^3-2-\left(3x^4-2x\right)\ln 3}{5\cdot 3^x}$$

Alguns exemples més combinant les propietats:

$$\left(\frac{3x^2\sin x}{\sqrt[3]{x}}\right)'=\frac{(3x^2\sin x)'\cdot\sqrt[3]{x}-\left(3x^2\sin x\right)\left(\sqrt[3]{x}\right)'}{(\sqrt[3]{x})^2}=\frac{(6x\sin x+3x^2\cos x)\cdot\sqrt[3]{x}-\left(3x^2\sin x\right)\frac{1}{3\sqrt[3]{x^2}}}{(\sqrt[3]{x})^2}=$$

$$\frac{\frac{(6x\sin x+3x^2\cos x)\cdot\sqrt[3]{x}\cdot 3\sqrt[3]{x^2}}{3\sqrt[3]{x^2}}-\frac{\left(3x^2\sin x\right)}{3\sqrt[3]{x^2}}}{\sqrt[3]{x^2}}=\frac{\frac{(6x\sin x+3x^2\cos x)\cdot 3\sqrt[3]{x^3}}{3\sqrt[3]{x^2}}-\frac{\left(3x^2\sin x\right)}{3\sqrt[3]{x^2}}}{\sqrt[3]{x^2}}=$$

$$\frac{\frac{(6x\sin x+3x^2\cos x)\cdot 3x-3x^2\sin x}{3\sqrt[3]{x^2}}}{\sqrt[3]{x^2}}=\frac{18x^2\sin x+9x^3\cos x-3x^2\sin x}{3\sqrt[3]{x^2}}:\frac{\sqrt[3]{x^2}}{1}=$$

$$\frac{15x^2\sin x+9x^3\cos x}{3\sqrt[3]{x^2}\cdot\sqrt[3]{x^2}}=\frac{3x(5x\sin x+3x^2\cos x)}{3\sqrt[3]{x^4}}=\frac{\cancel{3x}(5x\sin x+3x^2\cos x)}{\cancel{3x}\cdot\sqrt[3]{x}}=$$

$$\frac{5x\sin x+3x^2\cos x}{\sqrt[3]{x}}$$

$$\left(\frac{x^2-3x+\sqrt{x}}{2x}\right)'=\left(\frac{x^2}{2x}-\frac{3x}{2x}+\frac{\sqrt{x}}{2x}\right)'=\left(\frac{1}{2}x-\frac{3}{2}+\frac{1}{2}x^{\frac{1}{2}-1}\right)'=\left(\frac{1}{2}x-\frac{3}{2}+\frac{1}{2}x^{-\frac{1}{2}}\right)'=$$

$$\frac{1}{2}-0+\frac{1}{2}\cdot\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}=\frac{1}{2}-\frac{1}{4\sqrt{x^3}}=\frac{1}{2}-\frac{1}{4x\sqrt{x}}$$

$$\left(\frac{(5x^3-10x^4)\cdot\sin x}{e^x-1}\right)'=\frac{\left[(5x^3-10x^4)'\sin x+(5x^3-10x^4)(\sin x)'\right](e^x-1)-(5x^3-10x^4)\sin x(e^x-1)'}{\left(e^x-1\right)^2}=$$

$$\frac{\left[(15x^2-40x^3)\cdot\sin x+(5x^3-10x^4)\cos x\right](e^x-1)-(5x^3-10x^4)e^x\sin x}{\left(e^x-1\right)^2}$$

$$\left(5(x^2-3).\ln x-\frac{x+4}{x^2}arctgx\right)'=(5(x^2-3).\ln x)'-(\frac{x+4}{x^2}arctgx)'=$$

$$5\cdot((x^2-3).\ln x)'-(\frac{x+4}{x^2}arctgx)'=5\cdot((x^2-3)'.\ln x+(x^2-3)\left(\ln x\right)')$$

$$-\left[\left(\frac{x+4}{x^2}\right)'arctgx+\frac{x+4}{x^2}(arctgx)'\right]=$$

$$5\cdot(2x.\ln x+(x^2-3)\frac{1}{x})-\left[\frac{x^2-(x+4)\cdot 2x}{x^4}arctgx+\frac{x+4}{x^2}\cdot\frac{1}{1+x^2}\right]=$$

$$5\cdot(2x.\ln x+(x^2-3)\frac{1}{x})-\left[\frac{-x^2-8x}{x^4}arctgx+\frac{x+4}{x^2(1+x^2)}\right]=$$

$$10x\ln x+\frac{5x^2-15}{x}+\frac{8x+x^2}{x^2}arctgx-\frac{x+4}{x^2(1+x^2)}$$